K = a constant
ln = natural logarithm
n = integer
N = rate of rotation of impeller, 1/hr.
N_P = power number, $Pg_c/\rho N^3 d^5$, dimensionless
N_{Re} = agitator Reynolds number, $d^2 N\rho/\mu$, dimensionless
N_{Sc} = continuous-phase Schmidt number, $\mu_C/\rho_C D_C$, dimensionless
N_{Sh} = continuous-phase Sherwood number, $k_C T/D_C$, dimensionless
N_{toD} = over-all number of dispersed-phase transfer units, dimensionless
N_{We} = Weber number, $d^3 N^2 \rho_C/\sigma$, dimensionless
P = power, ft. lb./hr.
r = radius of a sphere, ft.
s = a constant
t_C = continuous-phase temperature, °F.
t_D = final average dispersed-phase temperature, °F.
$t_D{}^0$ = initial uniform dispersed-phase temperature, °F.
T = diameter of mixing vessel, ft.
T_0 = standard diameter of mixing vessel = 0.5 ft. or equivalent in other units.
v_C = rate of flow of continuous phase, cu. ft./hr.
v_D = rate of flow of dispersed phase, cu. ft./hr.
V = volume of liquid in the mixing vessel, cu. ft.
x = a constant, the mixing index, dimensionless
X = $k_C d_p/2D_D'$, dimensionless
Z = $2k_C\theta/d_p$, dimensionless
θ = average dispersed-phase holding time, hrs.
μ = viscosity of agitated mixture, lbs./ft. hr.
μ_C = viscosity of continuous phase, lbs./ft. hr.
μ_D = viscosity of dispersed phase, lbs./ft. hr.
ν_C = kinematic viscosity of continuous phase = μ_C/ρ_C, sq. ft./hr.
ν_D = kinematic viscosity of dispersed phase = μ_D/ρ_D, sq. ft./hr.
π = 3.1416
ρ = density of agitated mixture, lb./cu. ft.
ρ_C = density of continuous phase, lb./cu. ft.
ρ_D = density of dispersed phase, lb./cu. ft.
$\Delta\rho$ = difference in density of the two liquids, lb./cu. ft.
σ = interfacial tension, lbs. mass/sq. hr. = 28.7 (10^3)(dynes/cm.)
ϕ_C = volume fraction of continuous phase in the mixing vessel
ϕ_D = volume fraction of dispersed phase in the mixing vessel
ψ_n = eigen value

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Flow of Two-phase Carbon Dioxide Through Orifices

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Experimental flow rate data are presented for saturated liquid, saturated vapor, and two-phase liquid-vapor carbon dioxide through a convergent nozzle and a square-edged orifice. The data cover the range from the triple-point pressure to the critical pressure. Charts have been prepared for this complete range at critical flow. Results are also presented for subcritical flow.

The tests at various back pressures indicate that the saturated liquid behaved as a cold liquid without evaporation ahead of the throat.

Saturated vapor became supersaturated in the nozzle, and the vapor behaved as if no condensation occurred.

Equations are presented for the flow rates of saturated vapor, and two-phase mixtures in the critical flow region.

A Mollier (pressure-enthalpy) diagram is used to determine the flow rates of saturated vapor and two-phase mixtures where supersaturation takes place. In these cases, the lines of constant specific volume or density are extrapolated from the superheated region into the normal two-phase region to obtain values corrected for supersaturation.

Critical flow nozzles or orifices are used in many cases to discharge two-phase carbon dioxide from piping systems to the atmosphere. This investigation was undertaken in order to predict the effect of nozzle or orifice design and operating conditions on the flow rate.

Whenever a saturated or boiling liquid flows through a pipeline, the drop in pressure, caused by the flow friction loss, is accompanied by a drop in temperature, and the boiling or evaporation of the liquid to convert a fraction of it to vapor in order to maintain thermodynamic equilibrium. If the liquid is colder than the pipeline, the heat transferred to the liquid also causes the evaporation of part of the liquid. This formation of vapor results in a two-phase mixture of liquid and vapor.

Such a flow of a two-phase mixture of liquid and vapor occurs when liquid carbon dioxide is used for fire extinguishing purposes or for industrial work such as cooling cold chambers or material which is being processed.

HISTORICAL

The flow of saturated liquid and saturated vapor carbon dioxide through nozzles at a back pressure of 142 lb./sq. in. abs. was recently investigated (*10*).

The phenomenon of supersaturation of vapor was disclosed in 1870 by Lord

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Kelvin (*14*) and in 1883 by Aitken (*2*). Supersaturation of steam flowing through nozzles was discussed by Martin (*18*) in 1918, in 1922 by Stoney (*25*), and in 1924 by Foa (*9*).

The flow of saturated water through orifices has been investigated (*3-7, 13, 15, 19, 24, 26, 27*).

The metastable flow of Freon liquid and other refrigerants through orifices has been investigated (*16, 20*).

APPARATUS

The equipment used in this investigation was explained in detail by Hesson (*12*). The orifice and nozzle arrangements are shown in Figure 1. The orifice and nozzle consisted of a stainless steel disk 0.350 in. in diam. by 0.062 in. thick with a hole 0.0357 ± 0.0001 in. in diam. On one side of the disk the entrance to the hole was well rounded while on the other side the entrance was square edged. When used as a square-edged orifice the square-edged entrance was placed upstream and when used as a convergent nozzle the rounded entrance was placed upstream. The area of the hole was 0.001002 ± 0.000006 sq. in. The ratio of d/D for the inlet was 0.192. This value was small enough to give negligible nozzle or orifice approach velocity U_1, but was large enough so that no separation of vapor and liquid was evident. The ratio of d/D for the downstream pipe was 0.1393. Pressure taps were located immediately upstream and downstream of the orifice disk to determine P_1 and P_3, respectively.

Two-phase mixtures were obtained by throttling saturated liquid. Saturated vapor was obtained by bubbling vapor through saturated liquid. Saturated liquid was obtained by heating liquid in contact with the vapor.

The flow rates were determined by evaporating and metering, at low pressure, the carbon dioxide which was discharged by the nozzle or orifice. The metering was done by orifice plates which were calibrated by weighing carbon dioxide.

For water at room temperature the discharge coefficient of the nozzle was 1.00.

BASIC EQUATIONS

The following are the basic equations used to compute the flow rates through an ideal nozzle for negligible approach velocity U_1 (*22*).

$$Q = 60(A/144)(U_2/V_2) \qquad (1)$$

$$Q = 60(A/144)(1/V_2) \cdot \left[2g_c(144) \int_{P_2}^{P_1} V\,dP \right]^{\frac{1}{2}} \qquad (2)$$

$$Q = 60(A/144)(1/V_2) \cdot [2g_c J(-)\,\Delta H_{12}]^{\frac{1}{2}} \qquad (3)$$

For critical flow, $P_2 = P_t$, $U_2 = U_t$, and $V_2 = V_t$. For subcritical flow, $P_2 = P_3$.

SATURATED VAPOR

Previous investigators have found that saturated steam can flow through a nozzle or orifice in a supersaturated condition (*4, 18, 25*).

In this investigation it was found that saturated carbon dioxide vapor can flow through a nozzle or orifice in a supersaturated condition. When flowing in a supersaturated condition, it behaves as superheated vapor with an expansion coefficient n, defined by the equation:

$$PV^n = P_1 V_1^n.$$

When this equation is substituted into Equation (2) and the integration is performed, the results for the nozzle are,

$$Q = \frac{60A[2g_c/144]^{\frac{1}{2}}\{(P_1/V_1)[n/(n-1)][1-(P_2/P_1)^{(n-1)/n}]\}^{\frac{1}{2}}}{(P_2/P_1)^{-1/n}} \qquad (4)$$

When P_3 is greater than P_t, $P_2 = P_3$. When P_3 is less than P_t, $P_2 = P_t$, and critical flow results,

$$Q = 60A[2g_c/144]^{\frac{1}{2}}[P_1/V_1]^{\frac{1}{2}} \cdot [n/(n+1)]^{\frac{1}{2}}[2/(n+1)]^{1/(n-1)} \qquad (5)$$

and

$$(P_t/P_1) = (2/(n+1))^{n/(n-1)} \qquad (6)$$

Values of n for use in Equations (4), (5), and (6) were determined from the experimental results and are shown in Table 1. The experimental results for critical flow are shown in Figure 2.

TABLE 1. VALUES OF n AND P_t/P_1 FOR SATURATED VAPOR

Pressure, P_1 lb./sq. in. abs.	n	P_t/P_1
0 to 600	1.30	0.55
700	1.22	0.56
800	1.17	0.57
900	1.07	0.59
1000	.94	0.62

A Mollier (pressure-enthalpy) diagram can be used to determine, approximately, the nozzle flow rates for saturated vapor with supersatuation in the nozzle. Figure 6 shows a section of a Mollier diagram, in the vicinity of the saturated vapor line, which was prepared from data from the literature (*1, 23*). A correction for supersaturation is made by extrapolating the lines of constant volume (or density) from the superheated vapor region across the saturated vapor line into the two-phase region. In Figure 6, these extrapolated lines are shown dotted. The diagram is used in the usual way for saturated vapor (when thermodynamic equilibrium is assumed) excepting that the dotted lines are used for the lines of constant volume in the two-phase region. Values of $(-\Delta H_{12})$ and V_2 are substituted in Equation (3). For critical flow a trial-and-error solution is necessary. Table 2 shows values computed from Figure 6 and Equation (3) for critical flow. These values check with the experimental values.

It will be noted that one of the effects of supersaturation is to increase the density for an expansion to a given pressure. Thus the actual flow rate is greater than that computed from the usual Mollier diagram for thermodynamic equilibrium.

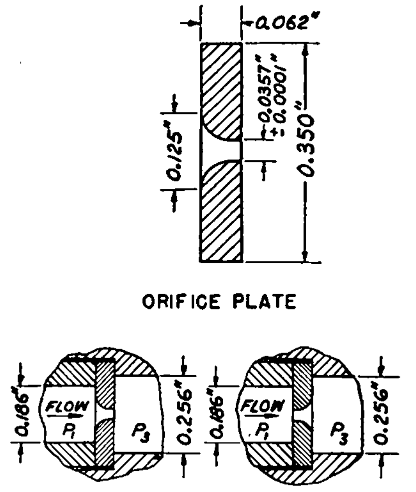
SATURATED LIQUID

Previous investigators have found that saturated water can flow through a nozzle in a superheated or metastable condition (*5, 6*). In boiling water at low back pressures, investigators have found (*5, 7*) that the boiling water behaves as cold water between the limits of the initial pressure and a lower pressure, which is equivalent to a critical throat pressure, at which point flashing occurs.

In this investigation, it was found that saturated carbon dioxide liquid can flow through a nozzle in a superheated or metastable condition and that it behaves as a nonvolatile liquid between the limits of the initial pressure and a lower pressure. On the assumption that the liquid is incompressible in Equation (2), the following results:

$$Q = 60A[2g_c/144]^{\frac{1}{2}} \cdot [(P_1 - P_2)/V_{f1}]^{\frac{1}{2}} \qquad (7)$$

This equation agrees with experimental results for values of P_2 from P_1 down to



ORIFICE PLATE

ARRANGEMENT FOR CONVERGING NOZZLE

ARRANGEMENT FOR SQUARE EDGE ORIFICE

Fig. 1. Orifice plate arrangement.

near the critical throat pressure P_t. Experimental results are shown in Figures 2 and 5.

TWO-PHASE MIXTURES

The two-phase mixtures were obtained by throttling saturated liquid. The experimental results for critical flow are shown in Figure 3 for the nozzle or rounded inlet orifice. Figure 4 shows the results for the square-edged inlet orifice. In these figures, lines of constant initial pressure P_0 are plotted. The orifice inlet pressure P_1 is the independent variable. The lines of constant x or liquid fraction by weight were determined by assuming isoenthalpic throttling or $\Delta H = 0$.

An unstable flow rate region is shown for the square-edged inlet orifice. This is discussed in detail by Hesson (*12*).

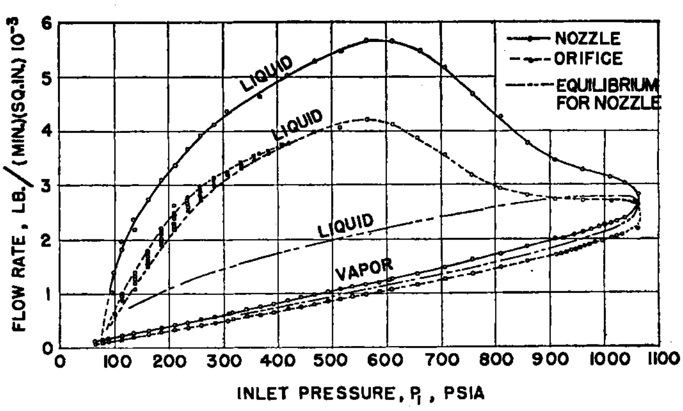


Fig. 2. Flow rates for saturated liquid and vapor carbon dioxide for nozzle and orifice for critical flow.

An equation which covers the saturated liquid, saturated vapor, and two-phase region for the nozzle was developed by making the following assumptions:

1. The initial vapor fraction of the mixture expands without heat or mass interchange with the remainder of the mixture according to the relationship $PV^n = P_1V_{g1}^n$.
2. The initial liquid fraction of the mixture may undergo some evaporation, but the vapor thus formed is considered part of the volume due to the initial liquid fraction, which expands according to the relationship, $PV^L = P_1V_{f1}^L$.
3. The velocity of both phases is the same.

The equation is:

$$Q = 5A\,\frac{\left[2g_c\,\frac{P_1}{V_{f^1}}\,\frac{1}{x}\right]^{\frac{1}{2}}\left[\frac{L}{L-1}\left\{1-\left(\frac{P}{P_1}\right)^{(L-1)/L}\right\}+\frac{1-x}{x}\,\frac{V_{g^1}}{V_{f^1}}\,\frac{n}{n-1}\left\{1-\left(\frac{P}{P_1}\right)^{(n-1)/n}\right\}\right]^{\frac{1}{2}}}{\left(\frac{P}{P_1}\right)^{-1/L}+\frac{1-x}{x}\,\frac{V_{g^1}}{V_{f^1}}\left(\frac{P}{P_1}\right)^{-1/n}} \qquad (8)$$

where n is a function of the initial pressure P_1, and L is a function of initial liquid fraction x. Values of n and L are shown in Tables 1 and 3 respectively.

For flow rates less than the critical, P is equal to the back pressure, P_3. For critical flow P/P_1 is evaluated for a maximum value of Q, in which case $P = P_t$, the throat pressure.

When the initial fraction of liquid is 1.0, $x = 1.0$ and the Equation (8) reduces to Equation (7) for saturated liquid.

When the initial fraction of liquid → 0, $x \rightarrow 0$, and the Equation (8) reduced to Equations (4*a*) and (5*a*) which further reduce to Equations (4) and (5) for saturated vapor,

$$Q = \frac{60A\,[2g_c/144]^{1/2}[1/(1-x)]^{1/2}\left\{\frac{P_1}{V_1}\,\frac{n}{n-1}\left[1-\left(\frac{P_2}{P_1}\right)^{(n-1)/n}\right]\right\}^{1/2}}{(P_2/P_1)^{-1/n}} \qquad (4a)$$

and for critical flow,

$$Q = 60A[2g_c/144]^{\frac{1}{2}}\left(\frac{P_1}{V_1}\right)^{\frac{1}{2}} \cdot[1/(1-x)]^{\frac{1}{2}}[n/(n+1)]^{\frac{1}{2}} \cdot[2/(n+1)]^{1/(n-1)} \qquad (5a)$$

TABLE 2. CRITICAL THROAT VALUES AND FLOW RATES FOR SATURATED VAPOR AT SUPERSATURATION CALCULATED FROM MOLLIER DIAGRAM

P_1 lb./sq. in. abs.	$-\Delta H_{12}$	V_t	U_t	U_t/V_t	P_t lb./sq. in. abs.	P_t/P_1	Q/A
770	7.50	0.15	612	4080	445	0.58	1700
550	9.28	0.24	680	2840	300	0.54	1180
345	9.32	0.42	682	1625	188	0.54	678
192	10.70	0.75	731	976	105	0.55	406

TABLE 3. VALUES OF L FOR EQUATION (8)

x	L
0 to 0.80	1.0
0.90	1.1
0.95	1.3
0.975	3.0

The Mollier (pressure-enthalpy) diagram of Figure 6 can be used to determine, approximately, the nozzle flow rates, corrected for vapor supersaturation, for two-phase mixtures with up to about 20% liquid by weight. In this case, the flow rates for saturated vapor are determined as discussed under saturated vapor. These values are then corrected by multiplying them by $[1/(1 - x)]^{1/2}$.

CONCLUSIONS

There is a paucity of data in the literature on flow of saturated liquid and two-phase carbon dioxide through nozzles and orifices. This investigation adds substantially to the available data.

Critical flow rates for saturated liquid, saturated vapor, and two-phase carbon dioxide for nozzles and orifices can be read from Figures 3 and 4.

For back pressures greater than the critical throat pressures, reference can be made to Figures 1 and 5, and to Equations (4), (7), and (8). For detailed information, reference can be made to the original work (*12*).

For systems other than carbon dioxide, the same general methods would apply. It is possible to estimate the flow of a two-phase system for small liquid fractions through a nozzle if a pressure-enthalpy diagram is available.

NOTATION

A = nozzle throat or orifice area, sq. in.
d = nozzle throat or orifice diameter, in.
D = nozzle or orifice pipe diameter, in.
g_c = conversion factor (lb.)(ft.)/(sec.)^2 (lb. force)
H = enthalpy, B.t.u./lb.
J = mechanical equivalent of heat, 778 ft. lb./B.t.u.
K = orifice coefficient, dimensionless
L = liquid expansion coefficient, dimensionless, where $PV^L = P_1V_{f_1}{}^L$
n = vapor expansion coefficient, dimensionless, where $PV^n = P_1V_{g_1}{}^n$

P = pressure, lb./sq. in. abs.
Q = flow rate, lb./min.
S = entropy, B.t.u./(lb.)(°F.)
U = velocity, ft./sec.
V = specific volume, cu. ft./lb.
x = liquid fraction by weight of two-phase mixture at P_1, dimensionless

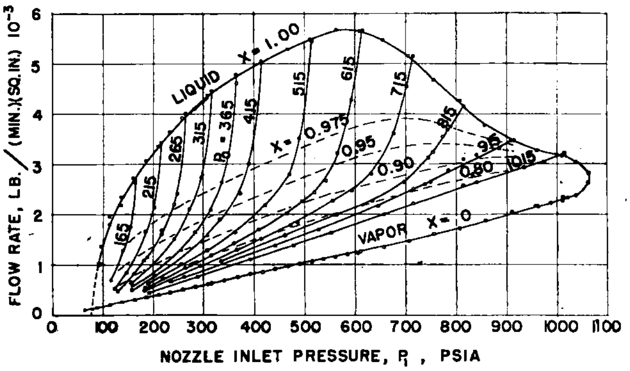


Fig. 3. Flow rates for two-phase (throttled saturated liquid) carbon dioxide for nozzle for critical flow (low back pressures).

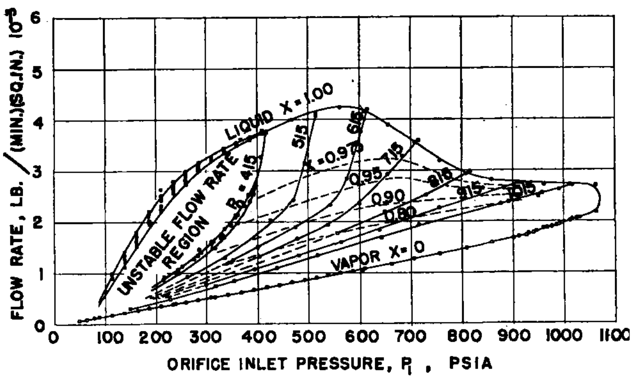


Fig. 4. Flow rates for two-phase (throttled saturated liquid) carbon dioxide for orifice at low back pressures.

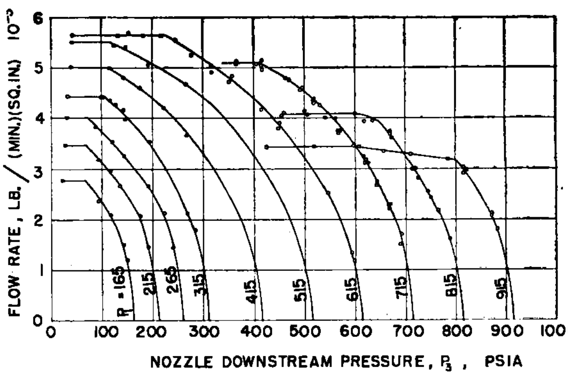


Fig. 5. Flow rates for saturated liquid carbon dioxide at various back pressures for nozzle.

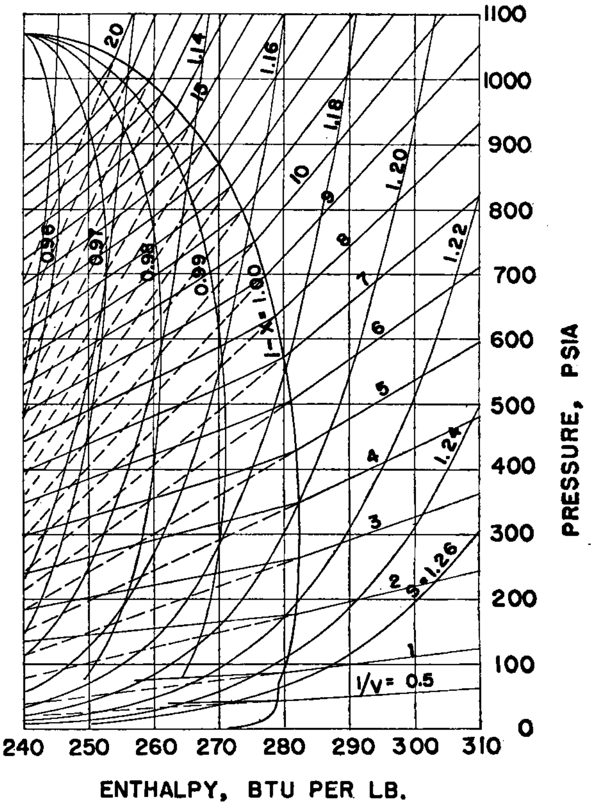


Fig. 6. Modified Mollier pressure-enthalpy diagram for carbon dioxide.

Subscripts

0 = storage condition
1 = nozzle inlet conditions
2 = nozzle throat conditions
3 = nozzle downstream conditions
f = saturated liquid
g = saturated vapor
t = nozzle throat conditions at critical flow rate

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